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# FINITE ELEMENT METHODS OF ANALYSIS FOR HIGH SPEED VISCOUS COMPRESSIBLE FLOWS

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## INTRODUCTION

In the past two reports we have described progress in the development of a two-step solution algorithm for the compressible Euler and Navier-Stokes equations on arbitrary unstructured meshes. In addition, we reported on initial experiments with adaptive mesh procedures, including mesh enrichment and mesh movement.

Over the past twelve months we have experimented with high resolution schemes on unstructured grids and have implemented an approach based upon flux corrected transport(FCT). The result was a scheme which gave much improved resolution of flow discontinuities but the convergence behaviour was not good for the solution of steady state problems. In addition we have written a triangulator for regions of arbitrary shape in 2D. This mesh generator has several novel features and has the ability to generate elements which are stretched in prescribed directions. Using this generator, we have proposed a new approach to mesh adaptation in which the mesh is regenerated at prescibed stages in the computation according to information which is provided by the solution on the current grid. This approach enables the user to have better control on the number of elements introduced at each stage and offers the best hope for mesh adaptation in 3D.

## FLUX CORRECTED TRANSPORT

The basic ideas of the FCT approach are well documented and our contribution has been to extend these ideas to the solution of equation systems on unstructured grids. The algorithm consists of the following steps:

(a) compute LEC, the low order element contributions from a low order scheme designed to give monotonic results for the problem being solved...we proposed the use of the basic two-step algorithm with a hefty amount of diffusion.

(b) compute HEC, the high order element contibutions....here we used the basic two-step algorithm with no artificial viscosity.

(c) define AEC, the anti-diffusive element contributions, as AEC=HEC-LEC.

(d) compute the updated low order solution for each grid point.

(e) limit the anti-diffusive element contributions in such a way that when thay are added to the updated low order solution the resulting solution exhibits no new extrema.

The results are extremely impressive for a single equation, but there are major problems in the limiting process when the approach is applied to systems as it is no longer clear exactly how the limiting process should be performed. We have shown that the best results come from applying the same limiter to each equation and to use an element limiter which is the minimum of those computed on the basis of density and pressure.

## TRIANGULAR MESH GENERATION

We experimented with triangular mesh generation for arbitrary 2D regions and produced an approach which was well-suited to our needs. This was a necessary experience building step before we began to think of the problem of generating meshes of tetrahedra in 3D. The triangulation requires information on the distribution of certain parameters which will define the local grid structure during the generation process. This is provided by interpolation of the parameters from user specified values given at the nodes of a coarse background grid which covers the region of interest. The generator uses the advancing front concept and places nodes and generates elements at the same time....the process ends when the front is empty. The final step is to perform certain cosmetics on the generated mesh e.g. smoothing, diagonal swaopping etc. The resulting meshes are generally of good quality and enable initial flow solutions to be obtained for any 2D problem.

## ADAPTIVE MESH REGENERATION

Adaptive refinement techniques based upon a-posteriori error estimators have been widely used to provide improved resolution of flow discontinuities. The common feature of these methods is that the refinement is accomplished by automatic subdivision of the cells or elements where the error estimate for the computed solution is above a certain threshold value. Despite the good numerical performance of these methods they do possess certain flaws...e.g. as the division of a cell produces cells of the same shape, the areas in the vicinity of 1D flow features are not refined in an efficient manner with the result that the number of cells employed increases rapidly as the computation advances. For this reason, such methods are likely to prove impractical in 3D. We are therefore proposing the use of an adaptive remeshing procedure in an attempt to reduce these difficulties. At each stage of the computation a completely new mesh is generated. The computed solution on the current mesh is used to determine, by means of an error analysis, the distribution of the optimum local grid parameters required by the triangulator described above. The current mesh then acts as a background grid and a new mesh is produced. The approach has been successfully used in the analysis of several different problems. The key to to the quality of the new grid is the error indication which is employed and this is an area which would benefit from further study...particulary for controlling the amount of derefining which is allowed and for handling problems involving thin boundary layers.

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